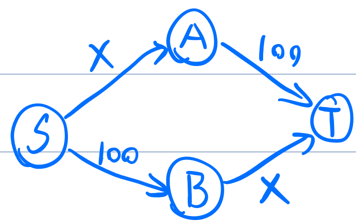
Today: Algorithmic Game Theory (AGT)

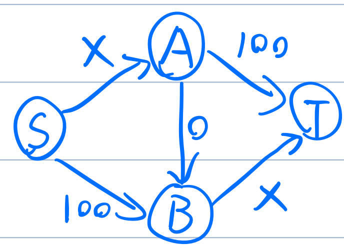
- Game theory × CS
- Algorithms in "strategic" environments

1. Traffic Networks

Imagine the following two traffic networks with costs.



Network 1

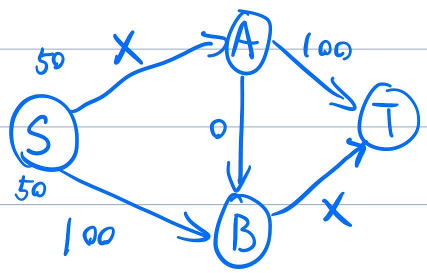


Network 2

For cost x, it means if there are x cars going from S to T, the cost is equal to x. Each driver minimizes their own cost, and we want to minimize average cost. Which network is better?

- If single driver, network 2.
- If 100 drivers, and we control where they go, network 2.
- If 100 drivers w/ free will what happens?

With this A→B shortcut, the first driver to take that path S→A→B→T only has cost 50+51=101 < 150. More drivers will take this route, when # of drivers reach 50, the cost of new path becomes



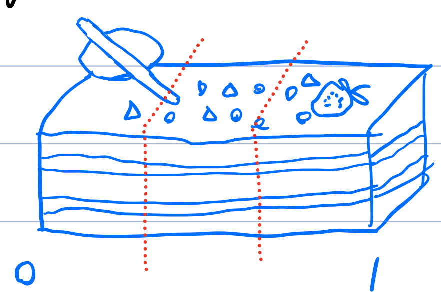
150, same as before, so no more improvements as opposed to S-A-T. But drivers in S-B-T has time 200, so they'll switch to S-A-B-T.

In the end, everyone goes S-A-B-T, average cost is 200! But without A-B, the average was only 150.

↑ Braess' Paradox

Key takeaway: when we want to say a network (algorithm/system) is "better", we need to explicitly model how people will use it.

2. Cake Cutting / Fair Division



Cutting a cake ($[0,1]$) and divide among n players.

Each player has a different evaluation of the cake $v_i : S \subseteq [0,1] \to [0,1]$.

e.g. $v_i([0,\frac{1}{3}])=1$

$v_i([\frac{1}{3},1])=0$

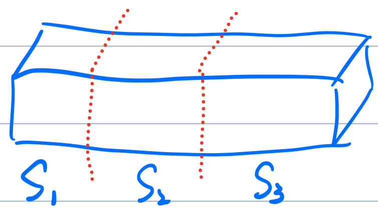
Envy-Free: An allocation $S_1, S_2, \dots, S_n$ to n players is envy-free if

$\forall i,j,\ v_i(S_i) \geq v_i(S_j)$

Two players: cut and choose

Player A cuts into 2 pieces, player B chooses first

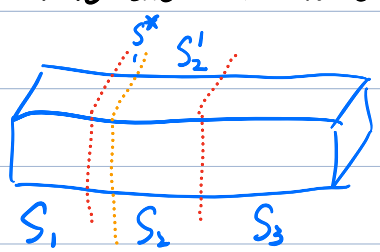
Three players?



Player A cuts into 3 pieces, let B&C chose?

What if they want the same piece? How to ensure fairness among B, C?

Idea: Have B "trim" its favorite piece, say S_2 to have the same value as its second favorite.



Now there are two "favorite pieces" for B.

Now let C choose between S_1, S_2', S_3.

Then B choose from the rest, and A gets the remaining.

A: ☺

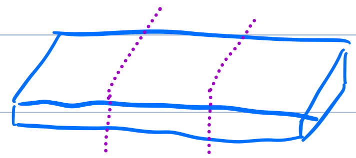
B: ☺

C: ☺

Done if we throw the trimming S^* away.

What if we want to keep it?

S^*



Let's say B gets S_2', now we let C cut.

(If C gets S_2', we let B cut.)

B chooses first, then A, then C.

B & C both happy, why?

Before allocating the trimming, both are happy.

& one gets to cut, and one gets to choose first.

A also happy.

A's cut in step 1 already better than entire S_2.

A's cut in step 1 as good as C's cut.

- - - - step 2 - - - - - - - - .

↑ Selfridge and Conway (~1960)

More players? 4 players, 203 cuts, by Aziz and Mackenzie (2016)

n players, $n^{n^{n^{n^{n^{n}}}}}$ cuts, by Aziz and Mackenzie (2016)